

# How to calculate ET using Priestley-Taylor with logger data and field weighing lysimeters

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Comments or suggestions are welcome to make this clearer/more user friendly: whittington.pete@gmail.com

## Background:

Evapotranspiration is typically the dominant water loss in wetlands, sometimes exceeding precipitation. As such, it is a very important component of the water balance to calculate. However, it is also quite difficult to calculate. One method commonly used in wetlands is the Priestley-Taylor (PT) method. Basically, you calculate what is called equilibrium evapotranspiration ( $E_{eq}$ ), based on the available energy, calculated from net radiation ( $Q^*$ ) minus the ground heat flux ( $Q_g$ ) as well as air temperature ( $T$ ) and some other constants. You then calibrate this value with actual evapotranspiration measured in the field (i.e. either at some previous time, or for short (e.g. daily) periods when you do have values. From this you can generate a complete daily (or hourly) data set based on logged values of the energy terms. While this can be done with sophisticated equipment and methods such as eddy covariance, it can also be done relatively simply using weighing lysimeters. Weighing lysimeters are essentially a block of soil in a container closed at the bottom and sides, open at the top and which is left for a specified period while it loses water to evapotranspiration. The specified period is often of irregular duration, and typically spotty with large gaps (for example when it is flooded by rain). For the periods deemed sufficiently accurate, the weight difference between the start and end of this irregular time period is then used to estimate water loss by knowing the density of water (to convert from mass to volume) and the open area of the lysimeter (to convert from volume to depth). You can then plot your calculated  $E_{eq}$  (for the same irregular period) against your field values of actual evaporation, and the slope of this line will be the calibration coefficient, known as the alpha value ( $\alpha$ ). Since you already have the equilibrium evaporation values, which are a continuous data set (from a data logger), you can then use them with the  $\alpha$  coefficient (that you just determined) to generate a continuous series of evaporation data. To state it simply, you're calibrating the met data with field data.

## Mathematics - Theory

The difficulty is how to compare the equilibrium vs. the actual evapotranspiration for irregular time periods. This document is designed to help you figure out what you need to do with your data to calculate your alpha value, but begins with a review of the mathematics.

Evaporation using the PT method is given by (*n.b.* For the sake of keeping all the terms in one place, I have used a table rather than the traditional in-text description):

$$E_t = \alpha \left( \frac{s}{s+\gamma} \right) \left( \frac{Q^* - Q_g}{L_v \rho} \right) \quad [1]$$

which means you will need to know  $s$ , the slope of the saturation vapour pressure curve, which is:

$$s = \frac{4098e^*}{(T+237.2)^2} \quad [2]$$

and thus you'll need  $e^*$ , the saturation vapour pressure,

$$e^* = 0.611 \cdot \exp \left( \frac{17.3 \cdot T}{T+237.3} \right) \quad [3]$$

and you will also need to know  $L_v$ , the latent heat of vaporization (which is used to convert the latent heat flux from an energy term to a water depth),

$$L_v = (2.501 - 0.002361 * T) \quad [4]$$

The meteorological variables ( $Q^*$ ,  $Q_g$  and  $T$ ) are determined using the appropriate instruments connected to a data logger and thus are not calculated. The psychrometric constant,  $\gamma$ , is a value that relates the partial pressures of water vapour in relation to all gases in air with changes in temperature. Using typical values in its calculation it becomes 0.066 kPa/°K (where K is degrees Kelvin).

Symbol	Term	Units (common)	Full units
$E_t$	Evapotranspiration	mm/day	
$\alpha$	alpha	-	
$s$	Slope of the vapour pressure curve	kPa/°C	Pa = kg/(m s <sup>2</sup> ); M/ (L T <sup>2</sup> )
$\gamma$	Psychrometric constant	kPa/°C	0.066 kPa/K
$Q^*$	Net Radiation	W/m <sup>2</sup>	1 Watt = Joule/Second= (kg m <sup>2</sup> )/s <sup>3</sup>
$Q_g$	Ground heat flux	W/m <sup>2</sup>	
$L_v$	Latent heat of vapourization	MJ/kg	
$T$	Temperature	C	
$e^*$	Saturation vapour pressure	kPa	

### Mathematics – practical

You'll note that the time factor in the units is typically per day, which proves problematic with erratic measurements of the lysimeters and sub-daily logger data (e.g., hourly). To solve the problem of unit conversions I have re-written equation 1 with all of the relevant unit conversions. Equations 2, 3 and 4 require NO unit conversion. This assumes temperature and energy ( $Q^*$  and  $Q_g$ ) are in °C and W/m<sup>2</sup>, respectively.

$$E_t = \alpha \left( \frac{s}{s + \gamma} \right) \left( \frac{Q^* - Q_g}{L_v * 1000000 * \rho} \right) * 1000 * 86400 = mm/day$$

The 1000000 in the denominator is to convert  $L_v$  from MJ to J.

The 1000 at the end is to convert m to mm.

The 86400 is to scale the instantaneous (per second) rate up to a daily value. The time is in seconds because the radiation is in Watts/m<sup>2</sup> and a Watt is a Joule/Second. *See important note below.*

To save you a lot of time and effort, you can just copy and paste the following formulae into Excel. This assumes your data starts in cell C5 (i.e., your first temperature value is in cell C5)

Column	What?	Formula
C	$T$	<i>none; raw data from logger</i>
D	$Q^*$	<i>none; raw data from logger</i>
E	$Q_g$	<i>none; raw data from logger</i>
F	$e^*$	= $(0.611 * \text{EXP}((17.3 * \text{C5}) / (\text{C5} + 237.3)))$
G	$s$	= $(4098 * \text{F5}) / ((237.3 + \text{C5})^2)$
H	$L_v$	= $(2.501 - (0.002361 * \text{C5}))$
I	$E_t$	= $((\text{G5} / (\text{G5} + 0.0662)) * ((\text{D5} - \text{E5}) / (\text{H5} * (1000000) * 1000))) * 1000 * \mathbf{86400}$

## The Time Factor

As mentioned earlier, the entire point to using the PT method is to calibrate the logger data with the field data. As such, you must compare the actual evaporation during the same time period. What this means is that rather than scaling up to a day (i.e., \*86400), you need to multiply by whatever time step your data are in. The table below shows some common logger outputs and their multiplier. Basically, you just need to figure out the number of seconds between your time steps.

Logger output	Multiplier (i.e., seconds)
10 minute	600
20 minute	1200
30 minute	1800
Hourly	3600
6 hour	21600
12 hour	43200
Daily	86400

## Calculating Alpha

On the next page is a sample data set using 10 minute data (and thus the multiplier is 600) and some hypothetical lysimeter data. Please note the absurdity of weighing a lysimeter every few hours and recording the mass to a gram; these data are purely for instructive purposes only. In reality you will likely be weighing your lysimeter, at the very most, daily.

Assume you weigh your lysimeter 5 times throughout this day and the masses are as follows:

Time of measurement	Mass (kg)	Evaporation* (mm)
21/07/2008 7:50	25	
21/07/2008 9:10	24.989	0.154
21/07/2008 10:10	24.971	0.260
21/07/2008 10:50	24.954	0.232
21/07/2008 11:30	24.931	0.330

\* nominally this is evaporation loss, but sometimes there is a gain because of rain, for which you will have to adjust based on rainfall measured simultaneously with a rain gauge, or you can ignore the data period with rain

At 9:10am the lysimeter weighs 0.011 kg less (or 11 grams) than the initial mass of 25 kg. Knowing that the density of water is  $\sim 1 \text{ g/cm}^3$  (and thus  $1 \text{ cm}^3$  of water weighs 1 gram), we can say that the lysimeter lost  $11 \text{ cm}^3$  of water. If the lysimeter bucket has a diameter of 30 cm then it has an area of  $\sim 707 \text{ cm}^2$  ( $\text{Area} = \pi r^2 = 3.14 * 15^2$ ). We can then convert the volume of water lost ( $11 \text{ cm}^3$ ) to a depth of water lost by dividing by the area, or  $11 \text{ cm}^3 / 707 \text{ cm}^2$ , which equals 0.0154 cm or 0.154 mm of evaporation between 7:50 and 9:10 am.

Repeat this for the subsequent time periods.

You now need to sum the equilibrium evaporation for the same time period. You can sort through the data manually, or you can automate it in Excel using a =SUMIF statement. Rather than summing the data between two measurement points, the =SUMIF statement works by summing all of the data to your measurement point and then subtracting the

sum of all of the data to the previous measurement point. The formula in cell O10 (see the spreadsheet below which illustrates the setup and data-requirements) would be:

=SUMIF(\$B\$5:\$B\$40,"<="&L10,\$I\$5:\$I\$40)-SUMIF(\$B\$5:\$B\$40,"<="&L9,\$I\$5:\$I\$40)

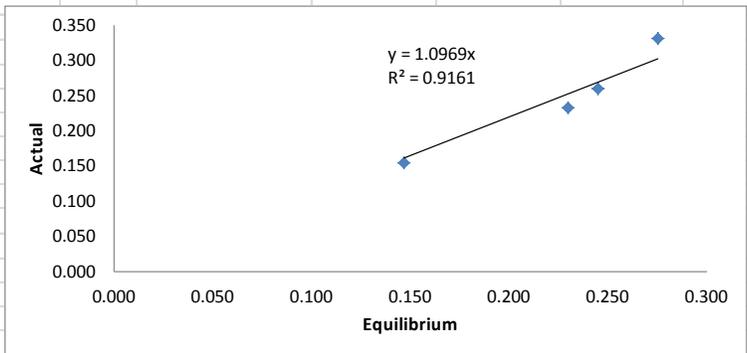
Which in English reads: sum all of the values in column I when the values in column B are less than or equal to the time in cell L10 (i.e., the second measurement period, 9:10am), then, subtract the sum of the values in column I when the values in column B are less than or equal to the time in cell L9 (i.e., the previous measurement time, 7:50am). This value is 0.147 mm, which is functionally the sum of the data from I17 to I24 (first greyed data).

Repeat this for all your measurement periods. If you have your data set-up as I do below, you can simply drag the formula in O10 down.

Note: the values in column I are the evaporation rates in mm/10 minute intervals.

Note also that you do not want to include the 7:50 data (i.e., cell I16, 0.006 mm/10 minute) in your sum, as this data point is for the preceding time (i.e, 7:40:00-7:49:59), which occurred *before* your measurement point.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
4	Date/Time	Temp	Q*	Qg	e*	s	Lv	Et						
5	21/07/2008 6:00	9.43	-53.69	13.81	1.184	0.080	2.479	-0.009				area of bucket	706.9	
6	21/07/2008 6:10	9.54	-50.94	13.81	1.192	0.080	2.478	-0.009						
7	21/07/2008 6:20	9.68	-45.88	13.81	1.204	0.081	2.478	-0.008						
8	21/07/2008 6:30	9.84	-36.73	13.81	1.217	0.082	2.478	-0.007						
9	21/07/2008 6:40	10.05	-29.52	13.81	1.234	0.083	2.477	-0.006			Time of measurement	Mass (kg)	Actual	mm/time step
10	21/07/2008 6:50	10.31	-23.83	13.81	1.256	0.084	2.477	-0.005			21/07/2008 7:50	25		Equilibrium
11	21/07/2008 7:00	10.53	-30.35	13.81	1.274	0.085	2.476	-0.006			21/07/2008 9:10	24.989	0.154	0.147
12	21/07/2008 7:10	10.69	-31.70	13.81	1.288	0.086	2.476	-0.006			21/07/2008 10:10	24.971	0.260	0.245
13	21/07/2008 7:20	10.90	12.43	13.81	1.306	0.087	2.475	0.000			21/07/2008 10:50	24.954	0.232	0.230
14	21/07/2008 7:30	11.14	26.80	13.81	1.327	0.088	2.475	0.002			21/07/2008 11:30	24.931	0.330	0.275
15	21/07/2008 7:40	11.38	41.62	13.81	1.349	0.089	2.474	0.004						
16	21/07/2008 7:50	11.76	57.59	13.81	1.383	0.091	2.473	0.006						
17	21/07/2008 8:00	11.98	72.48	13.81	1.403	0.093	2.473	0.008						
18	21/07/2008 8:10	12.32	91.20	13.81	1.435	0.094	2.472	0.011						
19	21/07/2008 8:20	12.59	109.10	13.81	1.461	0.096	2.471	0.014						
20	21/07/2008 8:30	13.00	127.60	13.81	1.501	0.098	2.470	0.017						
21	21/07/2008 8:40	13.45	147.50	13.81	1.545	0.101	2.469	0.020						
22	21/07/2008 8:50	13.84	167.60	13.81	1.585	0.103	2.468	0.023						
23	21/07/2008 9:00	14.36	187.20	13.81	1.640	0.106	2.467	0.026						
24	21/07/2008 9:10	14.93	205.80	13.81	1.701	0.110	2.466	0.029						
25	21/07/2008 9:20	15.48	225.90	13.81	1.763	0.113	2.464	0.033						
26	21/07/2008 9:30	15.89	245.90	13.81	1.810	0.116	2.463	0.036						
27	21/07/2008 9:40	16.34	264.60	13.81	1.862	0.119	2.462	0.039						
28	21/07/2008 9:50	16.89	281.50	13.81	1.929	0.122	2.461	0.042						
29	21/07/2008 10:00	17.22	300.70	13.81	1.970	0.125	2.460	0.046						
30	21/07/2008 10:10	17.74	321.10	13.81	2.035	0.128	2.459	0.049						
31	21/07/2008 10:20	18.21	338.90	13.81	2.097	0.132	2.458	0.053						
32	21/07/2008 10:30	18.75	356.90	13.81	2.169	0.136	2.457	0.056						
33	21/07/2008 10:40	19.20	370.50	13.81	2.231	0.139	2.456	0.059						
34	21/07/2008 10:50	19.61	383.50	13.81	2.288	0.142	2.455	0.062						
35	21/07/2008 11:00	19.97	402.70	13.81	2.340	0.145	2.454	0.065						
36	21/07/2008 11:10	20.13	417.30	13.81	2.363	0.146	2.453	0.068						
37	21/07/2008 11:20	20.47	429.00	13.81	2.414	0.149	2.453	0.070						
38	21/07/2008 11:30	20.77	435.90	13.81	2.459	0.151	2.452	0.072						
39	21/07/2008 11:40	21.21	445.20	13.81	2.526	0.155	2.451	0.074						
40	21/07/2008 11:50	21.35	458.80	13.81	2.548	0.156	2.451	0.077						



And finally... once you have done this, you should have two columns of evaporation data, Actual and Equilibrium (columns N and O), create an X-Y scatter plot (ensure Equilibrium is on the x-axis), add a trend-line (force intercept through 0) and equation, and voila, you have your alpha value (the m in  $y = mx$ ); in this case, 1.09. Now, multiply your equilibrium evaporation data (column I) by 1.09 (i.e., to complete equation 1).

## **Some other things**

This help file was designed to show you how to calculate the equilibrium evapotranspiration from logger data and compare it with your field (lysimeter) data for the same time periods, regress them, and determine your alpha value. In reality, determining your alpha value is much more difficult than I have led you to believe. (You won't get an  $r^2$  value of 0.93!) This is in large part due to rain and extended time periods between measurement points and field (weighting) errors. As mentioned in the Background section you will need to carefully go through your field data looking for good lysimeter data; you may end up throwing half of it away!

In addition, deriving 1 alpha value for your entire site/season is likely erroneous. Typically you should calculate alpha both spatially and temporally. That is between landscape types (lawns, hummocks, ridges, pools, forests etc.) and throughout the season (spring, summer, fall).

## **References**

While I have not referenced anything in the text above, below are some useful papers to read about evaporation.

Ingram, H.A.P., 1983. Hydrology. In: Gore, A.J.P. (Ed.), Ecosystems of the world. Vol 4A: Mires: Swamp, Bog, Fen and Moor. Elsevier Scientific Publishing Company, New York, pp. 67-158.

Price, J., 1991. Evaporation from a blanket bog in a foggy coastal environment. *Boundary-Layer Meteorology*, 57(4): 391-406.

Price, J.S., Maloney, D.A., 1994. Hydrology of a patterned bog-fen complex in southeastern Labrador, Canada. *Nordic Hydrology*, 25: 313-330.

Priestley, C.H.B., Taylor, R.J., 1972. On the assessment of surface heat flux and evaporation using large-scale parameters. *Monthly Weather Review*, 100(2): 81-92.